13-2 Verifying Trigonometric Identities

CCSS PRECISION Verify that each equation is an identity.

1. $\cot\theta + \tan\theta = \dfrac{\sec^2\theta}{\tan\theta}$

SOLUTION:

$$\cot\theta + \tan\theta \stackrel{?}{=} \frac{\sec^2\theta}{\tan\theta}$$
$$\cot\theta + \tan\theta \stackrel{?}{=} \frac{\tan^2\theta + 1}{\tan\theta}$$
$$\cot\theta + \tan\theta \stackrel{?}{=} \frac{\tan^2\theta}{\tan\theta} + \frac{1}{\tan\theta}$$
$$\cot\theta + \tan\theta = \tan\theta + \cot\theta \checkmark$$

2. $\cos^2\theta = (1 + \sin\theta)(1 - \sin\theta)$

SOLUTION:

$$\cos^2\theta \stackrel{?}{=} (1 + \sin\theta)(1 - \sin\theta)$$
$$\cos^2\theta \stackrel{?}{=} 1 - \sin^2\theta$$
$$\cos^2\theta = \cos^2\theta \checkmark$$

3. $\sin\theta = \dfrac{\sec\theta}{\tan\theta + \cot\theta}$

SOLUTION:

$$\sin\theta \stackrel{?}{=} \frac{\sec\theta}{\tan\theta + \cot\theta}$$
$$\sin\theta \stackrel{?}{=} \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$
$$\sin\theta \stackrel{?}{=} \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$$
$$\sin\theta \stackrel{?}{=} \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta\cos\theta}}$$
$$\sin\theta \stackrel{?}{=} \frac{1}{\cos\theta}\,\frac{\sin\theta\cos\theta}{1}$$
$$\sin\theta = \sin\theta \checkmark$$

4. $\tan^2\theta = \dfrac{1-\cos^2\theta}{\cos^2\theta}$

SOLUTION:

$$\tan^2\theta \stackrel{?}{=} \frac{1-\cos^2\theta}{\cos^2\theta}$$
$$\tan^2\theta \stackrel{?}{=} \frac{1-\cos^2\theta}{\cos^2\theta}$$
$$\tan^2\theta \stackrel{?}{=} \frac{\sin^2\theta}{\cos^2\theta}$$
$$\tan^2\theta = \tan^2\theta \checkmark$$

5. $\tan^2\theta\csc^2\theta = 1+\tan^2\theta$

SOLUTION:

$$\tan^2\theta\csc^2\theta \stackrel{?}{=} 1+\tan^2\theta$$
$$\frac{\sin^2\theta}{\cos^2\theta}\frac{1}{\sin^2\theta} \stackrel{?}{=} \sec^2\theta$$
$$\frac{1}{\cos^2\theta} \stackrel{?}{=} \sec^2\theta$$
$$\sec^2\theta = \sec^2\theta \checkmark$$

6. $\tan^2\theta = (\sec\theta+1)(\sec\theta-1)$

SOLUTION:

$$\tan^2\theta \stackrel{?}{=} (\sec\theta+1)(\sec\theta-1)$$
$$\tan^2\theta \stackrel{?}{=} \sec^2\theta-1$$
$$\tan^2\theta = \tan^2\theta \checkmark$$

7. **MULTIPLE CHOICE** Which expression can be used to form an identity with $\frac{\tan^2\theta+1}{\tan^2\theta}$?

A. $\sin^2\theta$
B. $\cos^2\theta$
C. $\tan^2\theta$
D. $\csc^2\theta$

SOLUTION:

$$\begin{aligned}\frac{\tan^2\theta+1}{\tan^2\theta} &= \frac{\sec^2\theta}{\tan^2\theta}\\ &= \frac{1}{\cos^2\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\\ &= \frac{1}{\sin^2\theta}\\ &= \csc^2\theta\end{aligned}$$

Option D is the correct answer.

Verify that each equation is an identity.

8. $\cos^2\theta+\tan^2\theta\cos^2\theta=1$

SOLUTION:

$$\begin{aligned}\cos^2\theta+\tan^2\theta\cos^2\theta &\stackrel{?}{=} 1\\ \cos^2\theta+\frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta &\stackrel{?}{=} 1\\ \cos^2\theta+\sin^2\theta &\stackrel{?}{=} 1\\ 1 &= 1\checkmark\end{aligned}$$

9. $\cot\theta(\cot\theta+\tan\theta)=\csc^2\theta$

SOLUTION:

$$\begin{aligned}\cot\theta(\cot\theta+\tan\theta) &\stackrel{?}{=} \csc^2\theta\\ \cot^2\theta+\cot\theta\tan\theta &\stackrel{?}{=} \csc^2\theta\\ \cot^2\theta+\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} &\stackrel{?}{=} \csc^2\theta\\ \cot^2\theta+1 &\stackrel{?}{=} \csc^2\theta\\ \csc^2\theta &= \csc^2\theta\checkmark\end{aligned}$$

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10. $1+\sec^2\theta\sin^2\theta=\sec^2\theta$

SOLUTION:

$$1+\sec^2\theta\sin^2\theta\overset{?}{=}\sec^2\theta$$
$$1+\frac{1}{\cos^2\theta}\cdot\sin^2\theta\overset{?}{=}\sec^2\theta$$
$$1+\tan^2\theta\overset{?}{=}\sec^2\theta$$
$$\sec^2\theta=\sec^2\theta\checkmark$$

11. $\sin\theta\sec\theta\cot\theta=1$

SOLUTION:

$$\sin\theta\sec\theta\cot\theta\overset{?}{=}1$$
$$\sin\theta\cdot\frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\overset{?}{=}1$$
$$1=1\checkmark$$

12. $\frac{1-\cos\theta}{1+\cos\theta}=\left(\csc\theta-\cot\theta\right)^2$

SOLUTION:

$$\frac{1-\cos\theta}{1+\cos\theta}\overset{?}{=}(\csc\theta-\cot\theta)^2$$
$$\frac{1-\cos\theta}{1+\cos\theta}\overset{?}{=}\csc^2\theta-2\cot\theta\csc\theta+\cot^2\theta$$
$$\frac{1-\cos\theta}{1+\cos\theta}\overset{?}{=}\frac{1}{\sin^2\theta}-2\cdot\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}+\frac{\cos^2\theta}{\sin^2\theta}$$
$$\frac{1-\cos\theta}{1+\cos\theta}\overset{?}{=}\frac{1}{\sin^2\theta}-\frac{2\cos\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}$$
$$\frac{1-\cos\theta}{1+\cos\theta}\overset{?}{=}\frac{1-2\cos\theta+\cos^2\theta}{\sin^2\theta}$$
$$\frac{1-\cos\theta}{1+\cos\theta}\overset{?}{=}\frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$$
$$\frac{1-\cos\theta}{1+\cos\theta}\overset{?}{=}\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$
$$\frac{1-\cos\theta}{1+\cos\theta}=\frac{1-\cos\theta}{1+\cos\theta}\checkmark$$

13. $\dfrac{1-2\cos^2\theta}{\sin\theta\cos\theta}=\tan\theta-\cot\theta$

SOLUTION:

$$\frac{1-2\cos^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\tan\theta-\cot\theta$$

$$\frac{\left(1-\cos^2\theta\right)-\cos^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\tan\theta-\cot\theta$$

$$\frac{\sin^2\theta-\cos^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\tan\theta-\cot\theta$$

$$\frac{\sin^2\theta}{\sin\theta\cos\theta}-\frac{\cos^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\tan\theta-\cot\theta$$

$$\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}\stackrel{?}{=}\tan\theta-\cot\theta$$

$$\tan\theta-\cot\theta=\tan\theta-\cot\theta\ \checkmark$$

14. $\tan\theta=\dfrac{\sec\theta}{\csc\theta}$

SOLUTION:

$$\tan\theta\stackrel{?}{=}\frac{\sec\theta}{\csc\theta}$$

$$\tan\theta\stackrel{?}{=}\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}$$

$$\tan\theta\stackrel{?}{=}\frac{\sin\theta}{\cos\theta}$$

$$\tan\theta=\tan\theta\checkmark$$

15. $\cos\theta=\sin\theta\cot\theta$

SOLUTION:

$$\cos\theta\stackrel{?}{=}\sin\theta\cot\theta$$

$$\cos\theta\stackrel{?}{=}\sin\theta\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$\cos\theta=\cos\theta\checkmark$$

16. $(\sin\theta - 1)(\tan\theta + \sec\theta) = -\cos\theta$

SOLUTION:

$$(\sin\theta - 1)(\tan\theta + \sec\theta) \stackrel{?}{=} -\cos\theta$$

$$\sin\theta\tan\theta + \sin\theta\sec\theta - \tan\theta - \sec\theta \stackrel{?}{=} -\cos\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} \stackrel{?}{=} -\cos\theta$$

$$\frac{\sin^2\theta}{\cos\theta} - \frac{1}{\cos\theta} \stackrel{?}{=} -\cos\theta$$

$$\frac{\sin^2\theta - 1}{\cos\theta} \stackrel{?}{=} -\cos\theta$$

$$\frac{-\cos^2\theta}{\cos\theta} \stackrel{?}{=} -\cos\theta$$

$$-\cos\theta = -\cos\theta \checkmark$$

17. $\cos\theta\cos(-\theta) - \sin\theta\sin(-\theta) = 1$

SOLUTION:

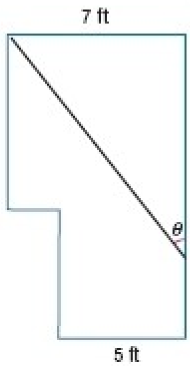
$$\cos\theta\cos(-\theta) - \sin\theta\sin(-\theta) \stackrel{?}{=} 1$$

$$\cos\theta\cos\theta - \sin\theta(-\sin\theta) \stackrel{?}{=} 1$$

$$\cos^2\theta + \sin^2\theta \stackrel{?}{=} 1$$

$$1 = 1 \checkmark$$

18. **LADDER** Some students derived an expression for the length of a ladder that, when carried flat, could fit around a corner from a 5-foot-wide hallway into a 7-foor-wide hallway, as shown. They determined that the maximum length ℓ of a ladder that would fit was given by $\ell(\theta) = \frac{7\sin\theta + 5\cos\theta}{\sin\theta\cos\theta}$. When their teacher worked the problem, she concluded that . $\ell(\theta) = 7\sec\theta + 5\csc\theta$ Are the two expressions equivalent?



SOLUTION:

$$\frac{7\sin\theta + 5\cos\theta}{\sin\theta\cos\theta} \stackrel{?}{=} 7\sec\theta + 5\csc\theta$$

$$\frac{7\sin\theta}{\sin\theta\cos\theta} + \frac{5\cos\theta}{\sin\theta\cos\theta} \stackrel{?}{=} 7\sec\theta + 5\csc\theta$$

$$\frac{7}{\cos\theta} + \frac{5}{\sin\theta} \stackrel{?}{=} 7\sec\theta + 5\csc\theta$$

$$7\sec\theta + 5\csc\theta = 7\sec\theta + 5\csc\theta \checkmark$$

Yes. They are equivalent.

Verify that each equation is an identity.

19. $(\sec\theta - \tan\theta) = \frac{1 - \sin\theta}{\cos\theta}$

SOLUTION:

$$\sec\theta - \tan\theta \stackrel{?}{=} \frac{1 - \sin\theta}{\cos\theta}$$

$$\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \stackrel{?}{=} \frac{1 - \sin\theta}{\cos\theta}$$

$$\frac{1 - \sin\theta}{\cos\theta} = \frac{1 - \sin\theta}{\cos\theta} \checkmark$$

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20. $\dfrac{1+\tan\theta}{\sin\theta+\cos\theta}=\sec\theta$

SOLUTION:

$$\frac{1+\tan\theta}{\sin\theta+\cos\theta}\stackrel{?}{=}\sec\theta$$
$$\frac{1+\frac{\sin\theta}{\cos\theta}}{\sin\theta+\cos\theta}\stackrel{?}{=}\sec\theta$$
$$\frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\sin\theta+\cos\theta}\stackrel{?}{=}\sec\theta$$
$$\frac{\cos\theta+\sin\theta}{\cos\theta}\frac{1}{\sin\theta+\cos\theta}\stackrel{?}{=}\sec\theta$$
$$\frac{1}{\cos\theta}\stackrel{?}{=}\sec\theta$$
$$\sec\theta=\sec\theta\checkmark$$

21. $\sec\theta\csc\theta=t\,\mathrm{an}\,\theta+\cot\theta$

SOLUTION:

$$\sec\theta\csc\theta\stackrel{?}{=}\tan\theta+\cot\theta$$
$$\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}\stackrel{?}{=}\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$
$$\frac{1}{\cos\theta\sin\theta}\stackrel{?}{=}\frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}$$
$$\frac{1}{\cos\theta\sin\theta}\stackrel{?}{=}\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$
$$\frac{1}{\cos\theta\sin\theta}=\frac{1}{\cos\theta\sin\theta}\checkmark$$

22. $\sin\theta+\cos\theta=\dfrac{2\sin^2\theta-1}{\sin\theta-\cos\theta}$

SOLUTION:

$$\sin\theta+\cos\theta\stackrel{?}{=}\frac{2\sin^2\theta-1}{\sin\theta-\cos\theta}$$
$$\sin\theta+\cos\theta\stackrel{?}{=}\frac{2\sin^2\theta-(\sin^2\theta+\cos^2\theta)}{\sin\theta-\cos\theta}$$
$$\sin\theta+\cos\theta\stackrel{?}{=}\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$$
$$\sin\theta+\cos\theta\stackrel{?}{=}\frac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$$
$$\sin\theta+\cos\theta=\sin\theta+\cos\theta\checkmark$$

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23. $(\sin\theta+\cos\theta)^2=\dfrac{2+\sec\theta\csc\theta}{\sec\theta\csc\theta}$

SOLUTION:

$$(\sin\theta+\cos\theta)^2 \stackrel{?}{=} \frac{2+\sec\theta\csc\theta}{\sec\theta\csc\theta}$$

$$(\sin\theta+\cos\theta)^2 \stackrel{?}{=} \frac{2+\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}}$$

$$(\sin\theta+\cos\theta)^2 \stackrel{?}{=} \left(2+\frac{1}{\cos\theta\sin\theta}\right)\cdot\frac{\cos\theta\sin\theta}{1}$$

$$(\sin\theta+\cos\theta)^2 \stackrel{?}{=} 2\cos\theta\sin\theta+1$$

$$(\sin\theta+\cos\theta)^2 \stackrel{?}{=} 2\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta$$

$$(\sin\theta+\cos\theta)^2 = (\sin\theta+\cos\theta)^2 \checkmark$$

24. $\dfrac{\cos\theta}{1-\sin\theta}=\dfrac{1+\sin\theta}{\cos\theta}$

SOLUTION:

$$\frac{\cos\theta}{1-\sin\theta}\stackrel{?}{=}\frac{1+\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta}{1-\sin\theta}\stackrel{?}{=}\frac{1+\sin\theta}{\cos\theta}\,\frac{1-\sin\theta}{1-\sin\theta}$$

$$\frac{\cos\theta}{1-\sin\theta}\stackrel{?}{=}\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$$

$$\frac{\cos\theta}{1-\sin\theta}\stackrel{?}{=}\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$\frac{\cos\theta}{1-\sin\theta}=\frac{\cos\theta}{1-\sin\theta}\checkmark$$

25. $\csc\theta - 1 = \dfrac{\cot 2\theta}{\csc\theta + 1}$

SOLUTION:

$$\csc\theta - 1 \stackrel{?}{=} \frac{\cot^2\theta}{\csc\theta + 1}$$
$$\csc\theta - 1 \stackrel{?}{=} \frac{\csc^2\theta - 1}{\csc\theta + 1}$$
$$\csc\theta - 1 \stackrel{?}{=} \frac{(\csc\theta - 1)(\csc\theta + 1)}{\csc\theta + 1}$$
$$\csc\theta - 1 = \csc\theta - 1 \checkmark$$

26. $\cos\theta\cot\theta = \csc\theta - \sin\theta$

SOLUTION:

$$\cos\theta\cot\theta \stackrel{?}{=} \csc\theta - \sin\theta$$
$$(\cos\theta)\frac{\cos\theta}{\sin\theta} \stackrel{?}{=} \frac{1}{\sin\theta} - \sin\theta$$
$$\frac{\cos^2\theta}{\sin\theta} \stackrel{?}{=} \frac{1}{\sin\theta} - \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\sin\theta}$$
$$\frac{\cos^2\theta}{\sin\theta} \stackrel{?}{=} \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$$
$$\frac{\cos^2\theta}{\sin\theta} \stackrel{?}{=} \frac{1 - \sin^2\theta}{\sin\theta}$$
$$\frac{\cos^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta} \checkmark$$

27. $\sin\theta\cos\theta\tan\theta + \cos^2\theta = 1$

SOLUTION:

$$\sin\theta\cos\theta\tan\theta + \cos^2\theta \stackrel{?}{=} 1$$
$$\sin\theta\cos\theta\frac{\sin\theta}{\cos\theta} + \cos^2\theta \stackrel{?}{=} 1$$
$$\sin^2\theta + \cos^2\theta \stackrel{?}{=} 1$$
$$1 = 1 \checkmark$$

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28. $(\csc\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

SOLUTION:

$$(\csc\theta - \cot\theta)^2 \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

$$\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2 \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

$$\frac{(1-\cos\theta)^2}{\sin^2\theta} \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

$$\frac{(1-\cos\theta)(1-\cos\theta)}{\sin^2\theta} \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

$$\frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

$$\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} \stackrel{?}{=} \frac{1-\cos\theta}{1+\cos\theta}$$

$$\frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\cos\theta}{1+\cos\theta} \checkmark$$

29. $\csc^2\theta = \cot^2\theta + \sin\theta\csc\theta$

SOLUTION:

$$\csc^2\theta \stackrel{?}{=} \cot^2\theta + \sin\theta\csc\theta$$

$$\csc^2\theta \stackrel{?}{=} \cot^2\theta + \sin\theta \cdot \frac{1}{\sin\theta}$$

$$\csc^2\theta \stackrel{?}{=} \cot^2\theta + 1$$

$$\csc^2\theta = \csc^2\theta \checkmark$$

30. $\frac{\sec\theta - \csc\theta}{\csc\theta\sec\theta} = \sin\theta - \cos\theta$

SOLUTION:

$$\frac{\sec\theta - \csc\theta}{\csc\theta\sec\theta} \stackrel{?}{=} \sin\theta - \cos\theta$$

$$\frac{\sec\theta}{\csc\theta\sec\theta} - \frac{\csc\theta}{\csc\theta\sec\theta} \stackrel{?}{=} \sin\theta - \cos\theta$$

$$\frac{1}{\csc\theta} - \frac{1}{\sec\theta} \stackrel{?}{=} \sin\theta - \cos\theta$$

$$\sin\theta - \cos\theta = \sin\theta - \cos\theta \checkmark$$

31. $\sin^2\theta + \cos^2\theta = \sec^2\theta - \tan^2\theta$

SOLUTION:

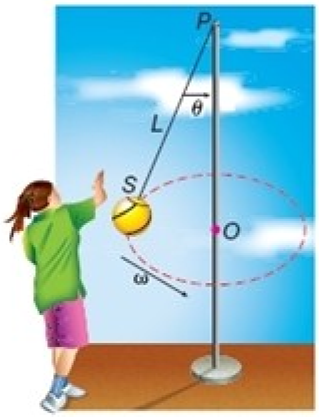
$$\begin{aligned}\sin^2\theta + \cos^2\theta &\stackrel{?}{=} \sec^2\theta - \tan^2\theta \\ 1 &\stackrel{?}{=} \tan^2\theta + 1 - \tan^2\theta \\ 1 &= 1 \checkmark\end{aligned}$$

32. $\sec\theta - \cos\theta = \tan\theta\sin\theta$

SOLUTION:

$$\begin{aligned}\sec\theta - \cos\theta &\stackrel{?}{=} \tan\theta\sin\theta \\ \frac{1}{\cos\theta} - \cos\theta &\stackrel{?}{=} \tan\theta\sin\theta \\ \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} &\stackrel{?}{=} \tan\theta\sin\theta \\ \frac{1-\cos^2\theta}{\cos\theta} &\stackrel{?}{=} \tan\theta\sin\theta \\ \frac{\sin^2\theta}{\cos\theta} &\stackrel{?}{=} \tan\theta\sin\theta \\ \left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta &\stackrel{?}{=} \tan\theta\sin\theta \\ \tan\theta\sin\theta &= \tan\theta\sin\theta \checkmark\end{aligned}$$

33. **CCSS SENSE-MAKING** The diagram below represents a game of tetherball. As the ball rotates around the pole, a conical surface is swept out by line segment $\overline{SP}$. A formula for the relationship between the length L of the string and the angle that the string makes with the pole is given by the equation $L = \frac{g \sec\theta}{\omega^2}$. Is $L = \frac{g \tan\theta}{\omega^2 \sin\theta}$ also an equation for the relationship between L and θ?



SOLUTION:

$$\frac{g \sec\theta}{\omega^2} \stackrel{?}{=} \frac{g \tan\theta}{\omega^2 \sin\theta}$$

$$\frac{g \sec\theta}{\omega^2} \stackrel{?}{=} \frac{g \frac{\sin\theta}{\cos\theta}}{\omega^2 \sin\theta}$$

$$\frac{g \sec\theta}{\omega^2} \stackrel{?}{=} \frac{g}{\omega^2 \cos\theta}$$

$$\frac{g \sec\theta}{\omega^2} = \frac{g \sec\theta}{\omega^2} \checkmark$$

Yes. $L = \frac{g \tan\theta}{\omega^2 \sin\theta}$ is also an equation for the relationship between L and θ.

34. **RUNNING** A portion of a racetrack has the shape of a circular arc with a radius of 16.7 meters. As a runner races along the arc, the sine of her angle of incline θ is found to be $\frac{1}{4}$. Find the speed of the runner. Use the Angle of Incline Formula given at the beginning of the lesson, $\tan\theta = \frac{v^2}{gR}$, where g = 9.8 and R is the radius. (Hint : Find cos θ first.)

SOLUTION:

Given, g = 9.8, R = 16.7 meters, $\sin\theta = \frac{1}{4}$.

Find cos θ.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \left(\frac{1}{4}\right)^2 + \cos^2\theta &= 1 \\ \cos\theta &= \sqrt{1 - \frac{1}{16}} \\ &= \frac{\sqrt{15}}{4} \end{aligned}$$

Find tan θ.

$$\begin{aligned} \tan\theta &= \frac{\sin\theta}{\cos\theta} \\ &= \frac{1/4}{\sqrt{15}/4} \\ &= \frac{1}{\sqrt{15}} \end{aligned}$$

Substitute the values of g, R and $\tan\theta$ in the equation $\tan\theta = \frac{v^2}{gR}$ and solve for v.

$$\begin{aligned} \tan\theta &= \frac{v^2}{gR} \\ \frac{1}{\sqrt{15}} &= \frac{v^2}{9.8(16.7)} \\ v &= \sqrt{\frac{9.8(16.7)}{\sqrt{15}}} \\ &= 6.5 \text{ m/s} \end{aligned}$$

When simplified, would the expression be equal to 1 or -1?

35. $\cot(-\theta)\tan(-\theta)$

SOLUTION:

$$\begin{aligned}\cot(-\theta)\tan(-\theta) &= (-\cot\theta)(-\tan\theta)\\ &= \frac{1}{-\tan\theta}\cdot(-\tan\theta)\\ &= 1\end{aligned}$$

36. $\sin\theta\csc(-\theta)$

SOLUTION:

$$\begin{aligned}\sin\theta\csc(-\theta) &= \frac{\sin\theta}{\sin(-\theta)}\\ &= \frac{\sin\theta}{-\sin\theta}\\ &= -1\end{aligned}$$

37. $\sin^2(-\theta)+\cos^2(-\theta)$

SOLUTION:

$$\begin{aligned}\sin^2(-\theta)+\cos^2(-\theta) &= \sin^2\theta+\cos^2\theta\\ &= 1\end{aligned}$$

38. $\sec(-\theta)\cos(-\theta)$

SOLUTION:

$$\begin{aligned}\sec(-\theta)\cos(-\theta) &= \frac{1}{\cos(-\theta)}\cdot\cos(-\theta)\\ &= \frac{1}{\cos\theta}\cdot\cos\theta\\ &= 1\end{aligned}$$

39. $\sec^2(-\theta)-\tan^2(-\theta)$

SOLUTION:

$$\begin{aligned}\sec^2(-\theta)-\tan^2(-\theta) &= \frac{1}{\cos^2(-\theta)}-\frac{\sin^2(-\theta)}{\cos^2(-\theta)}\\ &= \frac{1}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}\\ &= \frac{1-\sin^2\theta}{\cos^2\theta}\\ &= \frac{\cos^2\theta}{\cos^2\theta}\\ &= 1\end{aligned}$$

40. $\cot(-\theta)\cot\left(\frac{\pi}{2}-\theta\right)$

SOLUTION:

$$\begin{aligned}\cot(-\theta)\cot\left(\frac{\pi}{2}-\theta\right) &= -\cot\theta\cdot\tan\theta \\ &= -\frac{\tan\theta}{\tan\theta} \\ &= -1\end{aligned}$$

Simplify the expression to either a constant or a basic trigonometric function.

41. $\dfrac{\tan\left(\frac{\pi}{2}-\theta\right)\csc\theta}{\csc^2\theta}$

SOLUTION:

$$\begin{aligned}\frac{\tan\left(\frac{\pi}{2}-\theta\right)\csc\theta}{\csc^2\theta} &= \frac{\cot\theta\csc\theta}{\csc^2\theta} \\ &= \frac{\cot\theta}{\csc\theta} \\ &= \frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1} \\ &= \cos\theta\end{aligned}$$

42. $\dfrac{1+\tan\theta}{1+\cot\theta}$

SOLUTION:

$$\begin{aligned}\frac{1+\tan\theta}{1+\cot\theta} &= \frac{1+\tan\theta}{1+\frac{1}{\tan\theta}} \\ &= \frac{1+\tan\theta}{\frac{\tan\theta+1}{\tan\theta}} \\ &= \frac{\tan\theta(1+\tan\theta)}{1+\tan\theta} \\ &= \tan\theta\end{aligned}$$

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43. $\left(\sec^2\theta+\csc^2\theta\right)-\left(\tan^2\theta+\cot^2\theta\right)$

SOLUTION:

$$\begin{aligned}&\left(\sec^2\theta+\csc^2\theta\right)-\left(\tan^2\theta+\cot^2\theta\right)\\&=\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)-\left(\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}\right)\\&=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}-\frac{\sin^4\theta+\cos^4\theta}{\cos^2\theta\sin^2\theta}\\&=\frac{\sin^2\theta+\cos^2\theta-\sin^4\theta-\cos^4\theta}{\sin^2\theta\cos^2\theta}\\&=\frac{\sin^2\theta\left(1-\sin^2\theta\right)+\cos^2\theta\left(1-\cos^2\theta\right)}{\sin^2\theta\cos^2\theta}\\&=\frac{\sin^2\theta\cos^2\theta+\cos^2\theta\sin^2\theta}{\sin^2\theta\cos^2\theta}\\&=\frac{2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}\\&=2\end{aligned}$$

44. $\dfrac{\sec^2\theta-\tan^2\theta}{\cos^2 x+\sin^2 x}$

SOLUTION:

$$\begin{aligned}\frac{\sec^2\theta-\tan^2\theta}{\cos^2 x+\sin^2 x}&=\frac{1}{1}\\&=1\end{aligned}$$

45. $\tan\theta\cos\theta$

SOLUTION:

$$\begin{aligned}\tan\theta\cos\theta&=\frac{\sin\theta}{\cos\theta}\cdot\cos\theta\\&=\sin\theta\end{aligned}$$

46. $\cot\theta\tan\theta$

SOLUTION:

$$\begin{aligned}\cot\theta\tan\theta&=\frac{1}{\tan\theta}\cdot\tan\theta\\&=1\end{aligned}$$

13-2 Verifying Trigonometric Identities

47. $\sec\theta\sin\left(\frac{\pi}{2}-\theta\right)$

SOLUTION:

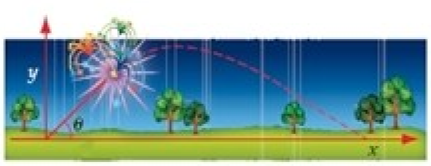
$$\begin{aligned}\sec\theta\sin\left(\frac{\pi}{2}-\theta\right) &= \sec\theta\cos\theta \\ &= \frac{1}{\cos\theta}\cdot\cos\theta \\ &= 1\end{aligned}$$

48. $\frac{1+\tan^2\theta}{\csc^2\theta}$

SOLUTION:

$$\begin{aligned}\frac{1+\tan^2\theta}{\csc^2\theta} &= \sin^2\theta\left(1+\tan^2\theta\right) \\ &= \sin^2\theta\left(\sec^2\theta\right) \\ &= \frac{\sin^2\theta}{\cos^2\theta} \\ &= \tan^2\theta\end{aligned}$$

49. **PHYSICS** When a firework is fired from the ground, its height y and horizontal displacement x are related by the equation $y=\frac{-gx^2}{2{v_0}^2\cos^2\theta}+\frac{x\sin\theta}{\cos\theta}$, where v_0 is the initial velocity of the projectile, θ is the angle at which it was fired, and g is the acceleration due to gravity. Rewrite this equation so that $\tan\theta$ is the only trigonometric function that appears in the equation.



SOLUTION:

$$\begin{aligned}y &= \frac{-gx^2}{2{v_0}^2\cos^2\theta}+\frac{x\sin\theta}{\cos\theta} \\ &= \frac{-gx^2}{2{v_0}^2}\sec^2\theta + x\tan\theta \\ &= \frac{-gx^2}{2{v_0}^2}\left(1+\tan^2\theta\right)+x\tan\theta\end{aligned}$$

50. **ELECTRONICS** When an alternating current of frequency f and peak current I_0 passes through a resistance R, then the power delivered to the resistance at time ` seconds is $P = I_0^2 R \sin^2 2\pi ft$.

a. Write an expression for the power in terms of $\cos^2 2\pi ft$.

b. Write an expression for the power in terms of $\csc^2 2\pi ft$.

SOLUTION:

a.

$$\begin{aligned} P &= I_0^2 R \sin^2(2\pi ft) \\ &= I_0^2 R\left(1 - \cos^2(2\pi ft)\right) \end{aligned}$$

b.

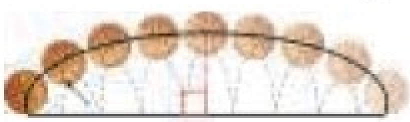
$$\begin{aligned} P &= I_0^2 R \sin^2(2\pi ft) \\ &= \frac{I_0^2 R}{\csc^2(2\pi ft)} \end{aligned}$$

51. **THROWING A BALL** In this problem, you will investigate the path of a ball represented by the equation $h = \frac{v_0^2 \sin^2 \theta}{2g}$, where θ is the measure of the angle between the ground and the path of the ball, v_0 is its initial velocity in meters per second, and g is the acceleration due to gravity. The value of g is 9.8 m./s^2.

a. If the initial velocity of the ball is 47 meters per second, find the height of the ball at 30° , 45°, 60°, and 90° . Round to the nearest tenth.

b. Graph the equation on a graphing calculator.

c. Show that the formula $h = \frac{v_0^2 \tan^2 \theta}{2g \sec^2 \theta}$ is equivalent to the one given above.



SOLUTION:

a. Substitute 9.8, 47 and 30° for g, v_0 and θ in the equation $h = \frac{v_0^2 \sin^2 \theta}{2g}$ and evaluate.

$$\begin{aligned} h &= \frac{v_0^2 \sin^2 \theta}{2g} \\ &= \frac{47^2 \sin^2 30°}{2(9.8)} \\ &\approx 28.2 \end{aligned}$$

Substitute 9.8, 47 and 45° for g, v_0 and θ in the equation $h = \frac{v_0^2 \sin^2 \theta}{2g}$ and evaluate.

$$h = \frac{v_0{}^2 \sin^2 \theta}{2g}$$
$$= \frac{47^2 \sin^2 45°}{2(9.8)}$$
$$\approx 56.4$$

Substitute 9.8, 47 and $60°$ for g, v_0 and θ in the equation $h = \frac{v_0{}^2 \sin^2 \theta}{2g}$ and evaluate.

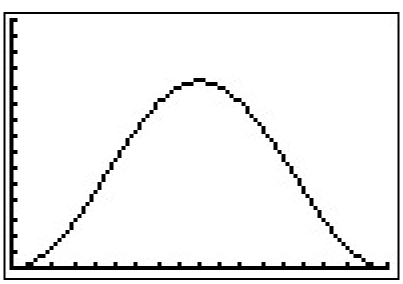
$$h = \frac{v_0{}^2 \sin^2 \theta}{2g}$$
$$= \frac{47^2 \sin^2 60°}{2(9.8)}$$
$$\approx 84.5$$

Substitute 9.8, 47 and $90°$ for g, v_0 and θ in the equation $h = \frac{v_0{}^2 \sin^2 \theta}{2g}$ and evaluate.

$$h = \frac{v_0{}^2 \sin^2 \theta}{2g}$$
$$= \frac{47^2 \sin^2 90°}{2(9.8)}$$
$$\approx 112.7$$

b. KEYSTROKES

Y= (SIN ALPHA [X]) X^2 ÷ (2 (9 . 8) GRAPH



[0,180] scl:10 by [0,150] scl:10

c.

$$\frac{v_0^{\,2}\tan^2\theta}{2g\sec^2\theta} \stackrel{?}{=} \frac{v_0^{\,2}\sin^2\theta}{2g}$$

$$\frac{v_0^{\,2}\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{2g\left(\frac{1}{\cos^2\theta}\right)} \stackrel{?}{=} \frac{v_0^{\,2}\sin^2\theta}{2g}$$

$$\frac{v_0^{\,2}\sin^2\theta}{2g} = \frac{v_0^{\,2}\sin^2\theta}{2g}$$

52. **CCSS ARGUMENTS** Identify the equation that does not belong with the other three. Explain your reasoning.

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\boxed{\sin^2\theta - \cos^2\theta = 2\sin^2\theta}$

$\boxed{1 + \cot^2\theta = \csc^2\theta}$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$

SOLUTION:

$\sin^2\theta - \cos^2\theta = 2\sin^2\theta$; the other three are Pythagorean identities, but this is not.

53. **CHALLENGE** Transform the right side of $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$ to show that $\tan^2\theta = \sec^2\theta - 1$.

SOLUTION:

$$\begin{aligned}\tan^2\theta &= \frac{\sin^2\theta}{\cos^2\theta}\\ &= \frac{1-\cos^2\theta}{\cos^2\theta}\\ &= \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}\\ &= \sec^2\theta - 1\end{aligned}$$

54. **WRITING IN MATH** Explain why you cannot square each side of an equation when verifying a trigonometric identity.

SOLUTION:

The properties of equality do not apply to identities as they do with equation. Do not perform operations to the quantities from each side of an unverified identity.

55. **REASONING** Explain why $\sin^2\theta + \cos^2\theta = 1$ is an identity, but $\sin\theta = \sqrt{1-\cos\theta}$ is not.

SOLUTION:

Sample answer: counterexample $45°$, $30°$

56. **WRITE A QUESTION** A classmate is having trouble trying to verify a trigonometric identity involving multiple trigonometric functions to multiple degrees. Write a question to help her work through the problem.

SOLUTION:

Sample answer: Have you tried using the most common identity, $\sin^2\sigma + \cos^2\sigma = 1$, to simplify?

57. **WRITING IN MATH** Why do you think expressions in trigonometric identities are often rewritten in terms of sine and cosine?

SOLUTION:

Sample answer: Sine and cosine are the trigonometric functions with which most people are familiar, and all trigonometric expressions can be written in terms of sine and cosine. Also, by rewriting complex trigonometric expressions in terms of sine and cosine it may be easier to perform operations and to apply trigonometric properties.

58. **CHALLENGE** Let $x = \frac{1}{2}\tan\theta, \text{where} -\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Write $f(x) = \frac{x}{\sqrt{1+4x^2}}$ in terms of a single trigonometric function of θ.

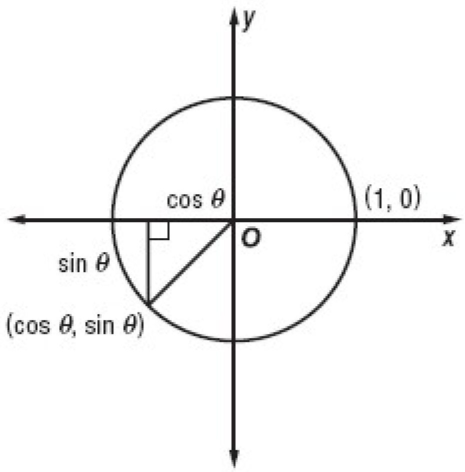
SOLUTION:

$$
\begin{aligned}
f(x) &= \frac{x}{\sqrt{1+4x^2}} \\
&= \frac{\frac{1}{2}\tan\theta}{\sqrt{1+4\left(\frac{1}{2}\tan\theta\right)^2}} \\
&= \frac{\tan\theta}{2\sqrt{1+\tan^2\theta}} \\
&= \frac{\tan\theta}{2\sqrt{\sec^2\theta}} \\
&= \frac{\tan\theta}{2\sec\theta} \\
f(x) &= \frac{\sin\theta}{2}
\end{aligned}
$$

59. **REASONING** Justify the three basic Pythagorean identities

SOLUTION:

Using the unit circle and the Pythagorean Theorem, we can justify $\cos^2\theta + \sin^2\theta = 1$.



If we divide each term of the identity $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$, we can justify $1 + \tan^2\theta = \sec^2\theta$.

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$
$$1 + \tan^2\theta = \sec^2\theta$$

If we divide each term of the identity $\cos^2\theta + \sin^2\theta = 1$ by $\sin^2\theta$, we can justify $\cot^2\theta + 1 = \csc^2\theta$.

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$
$$\cot^2\theta + 1 = \csc^2\theta$$

60. **SAT/ACT** A small business owner must hire seasonal workers as the need arises. The following list shows the number of employees hired monthly for a 5-month period.
5,14,6,8,12
If the mean of these data is 9, what is the population standard deviation for these data? (Round your answer to the nearest tenth.)
A 3.5
B 3.9
C 5.7
D 8.6
E 12.3

SOLUTION:

The population standard deviation is $s = \sqrt{\frac{\sum_{k=1}^{n}\left(x_n - \bar{x}\right)^2}{n}}$.

$$\sigma = \sqrt{\frac{(5-9)^2 + (14-9)^2 + (6-9)^2 + (8-9)^2 + (12-9)^2}{5}}$$
$$\approx 3.5$$

Therefore, option A is the correct answer.

61. Find the center and radius of the circle with equation $(x-4)^2 + y^2 - 16 = 0$.
F. $C(-4, 0)$; $r = 4$ units
G. $C(-4, 0)$; $r = 16$ units
H. $C(4, 0)$; $r = 4$ units
J. $C(4, 0)$; $r = 16$ units

SOLUTION:

$$(x-4)^2 + y^2 - 16 = 0$$
$$(x-4)^2 + y^2 = 16$$
$$(x-4)^2 + y^2 = 4^2$$

The center and the radius of the circle is (4, 0) and 4.
Therefore, option H is the correct answer.

62. **GEOMETRY** The perimeter of a right triangle is 36 inches. Twice the length of the longer leg minus twice the length of the shorter leg is 6 inches. What are the lengths of all three sides?
A. 3 in., 4 in., 5 in.
B. 6 in., 8 in., 10 in.
C. 9 in., 12 in., 15 in.
D. 12 in., 16 in., 20 in.

SOLUTION:

$2(12) - 2(9) = 6$

Therefore, option C is the correct answer.

63. Simplify $128^{\frac{1}{4}}$.
F. $2\sqrt[4]{2}$
G. $2\sqrt[4]{8}$
H. 4
J. $4\sqrt[4]{2}$

SOLUTION:

$$\begin{aligned}128^{\frac{1}{4}} &= \left(2^7\right)^{\frac{1}{4}} \\ &= 2^{\frac{7}{4}} \\ &= 2 \cdot 2^{\frac{3}{4}} \\ &= 2\sqrt[4]{2^3} \\ &= 2\sqrt[4]{8}\end{aligned}$$

Option G is the correct answer.

Find the exact value of each expression.

64. $\tan\theta, \text{if } \cot\theta = 2; 0° \le \theta < 90°$

SOLUTION:

$$\cot\theta = 2$$

$$\frac{1}{\tan\theta} = 2$$

$$\tan\theta = \frac{1}{2}$$

65. $\sin\theta$, if $\cos\theta = \frac{2}{3}; 0° \le \theta < 90°$

SOLUTION:

$$\text{Given } \cos\theta = \frac{2}{3}.$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{2}{3}\right)^2 = 1$$

$$\sin^2\theta = \frac{5}{9}$$

$$\sin\theta = \pm\frac{\sqrt{5}}{3}$$

Since θ is in the first quadrant, sin θ is positive.

Thus, $\sin\theta = \frac{\sqrt{5}}{3}$.

66. $\csc\theta$, if $\cos\theta = -\frac{3}{5}; 90° < \theta < 180°$

SOLUTION:

$$\text{Given } \cos\theta = -\frac{3}{5}.$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{3}{5}\right)^2 = 1$$

$$\sin^2\theta = \frac{16}{25}$$

$$\sin\theta = \pm\frac{4}{5}$$

$$\csc\theta = \pm\frac{5}{4}$$

Since θ is in the second quadrant, $\csc\theta$ is positive.

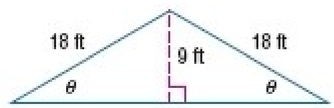
Thus, $\csc\theta = \frac{5}{4}$.

67. $\cos\theta$, if $\sec\theta = \frac{5}{3}$; $270° < \theta < 360°$

SOLUTION:

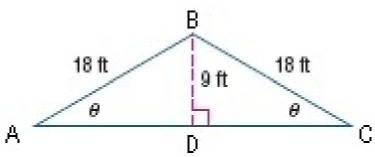
$$\text{Given } \sec\theta = \frac{5}{3}$$
$$\frac{1}{\cos\theta} = \frac{5}{3}$$
$$\cos = \frac{3}{5}$$

68. **ARCHITECTURE** The support for a roof is shaped like two right triangles, as shown in the figure. Find θ.



SOLUTION:

Name the vertices.



Consider the triangle ABD.

$$\sin\theta = \frac{9}{18}$$
$$= \frac{1}{2}$$
$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$
$$= 30°$$

69. **FAST FOOD** The table shows the probability distribution for value meals ordered at a fast food restaurant on Saturday mornings. Use this information to determine the expected value of the meals ordered.

Value Meals Ordered				
Meals	\$3	\$4	\$5	\$6
Probability	0.5	0.2	0.1	0.2

SOLUTION:

List meal price X along with the corresponding relative frequency $P(X)$. Find $X \cdot P(X)$. Then find the sum of those values.

X	$P(X)$	$X \cdot P(X)$
\$3	0.5	\$1.50
\$4	0.2	\$0.80
\$5	0.1	\$0.50
\$6	0.2	\$1.20
Total		\$4.00

The expected value is \$4.

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbolas with the given equations. Then graph the hyperbola.

70. $\frac{y^2}{18} - \frac{x^2}{20} = 1$

SOLUTION:

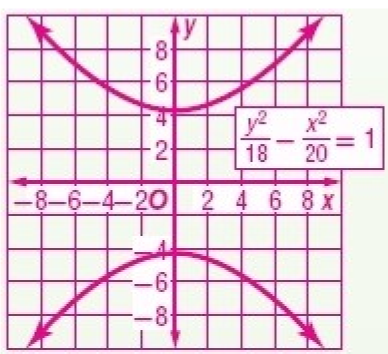
$a = \sqrt{18}, b = \sqrt{20}, h = 0$ and $k = 0$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{18 + 20} \\ &= \sqrt{38} \end{aligned}$$

The vertices of the hyperbola are $\left(0, \pm\sqrt{18}\right)$ or $\left(0, \pm 3\sqrt{2}\right)$.

The foci of the hyperbola are $\left(0, \pm\sqrt{38}\right)$.

The asymptotes for the hyperbola are $y = \pm\frac{\sqrt{18}}{\sqrt{20}}x = \pm\frac{3\sqrt{10}}{10}x$



71. $\frac{(y+6)^2}{20}-\frac{(x-1)^2}{25}=1$

SOLUTION:

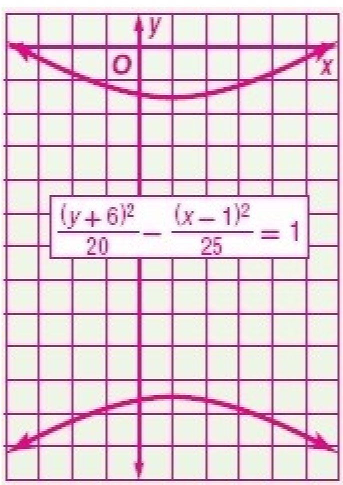
$a=\sqrt{20}$, $b=5$, $h=1$ and $k=-6$

$$\begin{aligned}c&=\sqrt{a^2+b^2}\\&=\sqrt{20+25}\\&=3\sqrt{5}\end{aligned}$$

The vertices of the hyperbola are $\left(1,-6\pm2\sqrt{5}\right)$.

The foci of the hyperbola are $\left(1,-6\pm3\sqrt{5}\right)$.

The asymptotes for the hyperbola are $y+6=\pm\frac{2\sqrt{5}}{5}(x-1)$



72. $x^2 - 36y^2 = 36$

SOLUTION:

$$x^2 - 36y^2 = 36$$
$$\frac{x^2}{36} - \frac{y^2}{1} = 1$$

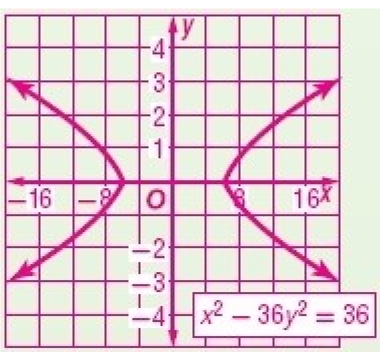
$a = 6, b = 1, h = 0$ and $k = 0$

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{36 + 1} \\ &= \sqrt{37} \end{aligned}$$

The vertices of the hyperbola are $(\pm 6, 0)$.

The foci of the hyperbola are $(\pm\sqrt{37}, 0)$.

The asymptotes for the hyperbola are $y = \pm\frac{1}{6}x$



Simplify.

73. $\frac{2+\sqrt{2}}{5-\sqrt{2}}$

SOLUTION:

$$\begin{aligned} \frac{2+\sqrt{2}}{5-\sqrt{2}} &= \frac{2+\sqrt{2}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} \\ &= \frac{(2+\sqrt{2})(5+\sqrt{2})}{5^2 - \sqrt{2}^2} \\ &= \frac{10 + 7\sqrt{2} + 2}{25 - 2} \\ &= \frac{12 + 7\sqrt{2}}{23} \end{aligned}$$

74. $\dfrac{x+1}{\sqrt{x^2-1}}$

SOLUTION:

$$\begin{aligned}\frac{x+1}{\sqrt{x^2-1}} &= \frac{x+1}{\sqrt{x^2-1}} \cdot \frac{\sqrt{x^2-1}}{\sqrt{x^2-1}} \\ &= \frac{(x+1)\sqrt{x^2-1}}{(x^2-1)} \\ &= \frac{(x+1)\sqrt{x^2-1}}{(x+1)(x-1)} \\ &= \frac{\sqrt{x^2-1}}{x-1}\end{aligned}$$

75. $\dfrac{x-1}{\sqrt{x}-1}$

SOLUTION:

$$\begin{aligned}\frac{x-1}{\sqrt{x}-1} &= \frac{x-1}{\sqrt{x}-1} \cdot \frac{-\sqrt{x}-1}{-\sqrt{x}-1} \\ &= \frac{-(x-1)\left(\sqrt{x}+1\right)}{-\left(\sqrt{x}^2-1^2\right)} \\ &= \frac{(x-1)\left(\sqrt{x}+1\right)}{x-1} \\ &= \sqrt{x}+1\end{aligned}$$

76. $\dfrac{-2-\sqrt{3}}{1+\sqrt{3}}$

SOLUTION:

$$
\begin{aligned}
\frac{-2-\sqrt{3}}{1+\sqrt{3}} &= \frac{-2-\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}\\
&= \frac{\left(-2-\sqrt{3}\right)\left(1-\sqrt{3}\right)}{1^2-\sqrt{3}^2}\\
&= \frac{-2-\sqrt{3}+2\sqrt{3}+3}{1-3}\\
&= \frac{-2+3+\sqrt{3}}{-2}\\
&= \frac{1+\sqrt{3}}{-2}\\
&= \frac{-1-\sqrt{3}}{2}
\end{aligned}
$$